Published by Institute of Physics Publishing for SISSA/ISAS

Received: *August 19, 2003*
Revised: *September 24, 2003*
Accepted: *September 24, 2003*

Cosmological solutions in multidimensional model with multiple exponential potential

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Abstract: A family of cosmological solutions with $(n+1)$ Ricci-flat spaces in the theory with several scalar fields and multiple exponential potential is obtained when coupling vectors in exponents obey certain relations. Two subclasses of solutions with power-law and exponential behaviour of scale factors are singled out. It is proved that power-law solutions may take place only when coupling vectors are linearly independent and exponential dependence occurs for linearly dependent set of coupling vectors. A subfamily of solutions with accelerated expansion is singled out. A generalized isotropization behaviours of certain classes of general solutions are found. In quantum case exact solutions to Wheeler-DeWitt equation are obtained and special "ground state" wave functions are considered.

Keywords: Sigma Models, Classical Theories of Gravity, Models of Quantum Gravity.

JHEP09(2003)059

Contents

1. Introduction

Recently, the discovery of the cosmic acceleration [1, 2] stimulated a lot of papers on multidimensional cosmology with the aim to explain this phenomenon using certain multidimensional models, e.g. those of superstring or supergravity origin (see [3]–[12], and references therein). These solutions usually deal with time-dependent scale factors of internal spaces, thus overcoming a "no-go" theorem for static (and compact) internal spaces [13]. It should be noted that certain part of publications does not deal with new exact solutions but use old ones (sometimes rediscovered or written in different parametrization).

A simple example of vacuum solution with acceleration was considered by Townsend and Wohlfarth in [3]. This is a $(4+n)$-dimensional solution to vacuum Einstein equations with n-dimensional internal space of negative curvature and 4-dimensional "our space" containing expanding 3-dimensional flat subspace. The solution from [3] is a special solution, in a special model with a very special choice of conformal frame. We note that it is a special case of more general vacuum solution from [14], describing the "evolution" of $(n-1)$ Ricci-flat spaces and one Einstein space of non-zero curvature. The solution from [14] was generalized to scalar-vacuum case in [15, 16, 17] and to the case of composite p-brane configurations in [19] (for non-composite case see also [18]). For a review see also [20]. We note that the solutions from [14, 16] may be also used for generation of special solutions with several curved factor spaces (e.g. from [43]) using a curvature-splitting trick [35] (when Einstein space of non-zero curvature is chosen as a product of several Einstein spaces). (For three integrable classes of vacuum cosmological solutions with two factor spaces of non-zero curvature in dimensions $D = 10, 11$ see [35].)

At present rather popular models are those with multiple exponential potential of the scalar fields (see, for example, [21, 23, 24, 25, 41, 42, 44] and refs. therein).

Such potentials also arise naturally in certain supergravitational models [22] and in sigma-models [26], related to configurations with p-branes.

Here we consider the D-dimensional model governed by the action

$$S_{act} = \int_M d^D z \sqrt{|g|} \{ \mathcal{R}[g] - h_{\alpha\beta} g^{MN} \partial_M \varphi^\alpha \partial_N \varphi^\beta - 2V_\varphi(\varphi) \} + S_{GH} \, , \tag{1.1}$$

with the scalar potential

$$V_\varphi(\varphi) = \sum_{s \in S} \Lambda_s \exp[2\lambda_s(\varphi)] \, . \tag{1.2}$$

Here $S_{\rm GH}$ is the standard Gibbons-Hawking boundary term [36].

The notations used here are the following ones:

- $\varphi = (\varphi^\alpha)$ is the vector from scalar fields in the space $\mathbb{R}^l$ with a metric determined by non-degenerate $l \times l$ matrix $(h_{\alpha\beta})$ with inverse one $(h^{\alpha\beta})$,

  $\alpha, \beta = 1, \dots, l$;

- Λ_s are constant terms,

  $s \in S$;

- λ_s is an 1-form on $\mathbb{R}^l$: $\lambda_s(\varphi) = \lambda_{s\alpha}\varphi^\alpha$; $\lambda_s^\alpha = h^{\alpha\beta}\lambda_{s\beta}$;

- $g = g_{MN} dz^M \otimes dz^N$ is the metric, $|g| = |\det(g_{MN})|$,

  M, N are world indices

  (that may be numerated by $1, \dots, D$);

- $i, j = 0, \dots, n$ are indices describing a chain of factor spaces;

  $A = i, \alpha$ and $B = j, \beta$ are minisuperspace indices

  (that may be numerated also by $0, \dots, n+l$).

In this article we obtain new (and general) families of classical and quantum "cosmological" solutions with vector coupling constants obeying

$$\lambda_{s\alpha}\lambda_{s'\beta}h^{\alpha\beta} = \frac{D-1}{D-2} \tag{1.3}$$

for $s \neq s'$ and

$$\lambda_{s\alpha}\lambda_{s\beta}h^{\alpha\beta} \neq \frac{D-1}{D-2}. \tag{1.4}$$

Here we find exact solutions with scale factors and scalar fields depending upon one variable u ("time"). We keep the parameter $w = \pm 1$ in the metric $g = w\mathrm{e}^{2\gamma(u)}du \otimes du + \ldots$ for a future consideration of static configurations [40]. They may be of interest in a context of "black hole" solutions with unusual asymptotics (see [27, 28, 29, 30, 12] and references therein). We single out two new subclasses of solutions with power-law and exponential behaviour of scale factors, e.g. those with accelerated expansion. A new result here is generalized isotropization behaviours of certain classes of classical solutions that give new examples of asymptotically accelerated expansion.

The paper is organized as follows. In section 2 the model is formulated. In section 3 a class of classical cosmological solutions corresponding to orthogonal U-vectors is presented. Section 4 deals with special solutions exhibiting power-law and exponential dependence of scale factors. Here we describe the solutions with accelerated expansion. A special subsection is devoted to a generalized isotropization. In section 5 quantum analogues of the classical solutions are considered. In appendix the equations of motion, corresponding to (1.1) are written (appendix A) and derivations of special solutions of power law and exponentional type are presented (appendix B and C); also a classification of vectors in euclidean space obeying relations (1.3) and (1.4) is done (appendix D).

We note that the classical and quantum solutions for cosmological constant case (i.e. for one term in potential with $\lambda_s = 0$) were considered earlier in [31] and [32] for vacuum and scalar-vacuum cases, respectively.

2. The model

Let

$$M = \mathbb{R}_{.} \times M_0 \times \ldots \times M_n \tag{2.1}$$

be a manifold equipped with the metric

$$g = w\mathrm{e}^{2\gamma(u)}du \otimes du + \sum_{i=0}^{n} \mathrm{e}^{2\phi^i(u)}g^i\,, \tag{2.2}$$

where $w = \pm 1$, $\mathbb{R}_{.}$ is open interval in $\mathbb{R}$, u is a distinguished coordinate; g^i is a Ricci-flat metric on d_i-dimensional manifold M_i:

$$\mathcal{R}_{m_i n_i}[g^i] = 0\,, \tag{2.3}$$

$d_i = \dim M_i$, $i = 0, \ldots, n$; $n \in \mathbb{N}$.

For dilatonic scalar fields we put

$$\varphi^{\alpha} = \varphi^{\alpha}(u)\,. \tag{2.4}$$

It may be verified that the equations of motion corresponding to (1.1) for the field configuration (2.2), (2.4) are equivalent to equations of motion for 1-dimensional σ-model with the action

$$S_{\sigma} = \frac{1}{2}\int du \mathcal{N}\Big\{G_{ij}\dot{\phi}^{i}\dot{\phi}^{j} + h_{\alpha\beta}\dot{\varphi}^{\alpha}\dot{\varphi}^{\beta} - 2\mathcal{N}^{-2}V\Big\}\,, \tag{2.5}$$

where $\dot{x} \equiv dx/du$,

$$V = -wV_{\varphi}(\varphi)\mathrm{e}^{2\gamma_0(\phi)} \tag{2.6}$$

is the potential (V_{φ} is defined in (1.2)) with

$$\gamma_0(\phi) \equiv \sum_{i=0}^{n} d_i\phi^{i}\,, \tag{2.7}$$

and

$$\mathcal{N} = \exp(\gamma_0(\phi) - \gamma) > 0 \tag{2.8}$$

is lapse function. Here

$$G_{ij} = d_i\delta_{ij} - d_i d_j\,, \qquad G^{ij} = \frac{\delta^{ij}}{d_i} + \frac{1}{2-D}\,, \tag{2.9}$$

$i, j = 0, \ldots, n$, are components of a gravitational part of minisupermetric and its dual [33].

2.1 Minisuperspace notations

In what follows we consider minisuperspace $\mathbb{R}^{n+1+l}$ with points

$$x \equiv (x^{A}) = (\phi^{i}, \varphi^{\alpha}) \tag{2.10}$$

equipped by minisuperspace metric $\bar{G} = \bar{G}_{AB}dx^{A} \otimes dx^{B}$ defined by the matrix and inverse one as follows:

$$(\bar{G}_{AB}) = \begin{pmatrix} G_{ij} & 0 \\ 0 & h_{\alpha\beta} \end{pmatrix}, \qquad (\bar{G}^{AB}) = \begin{pmatrix} G^{ij} & 0 \\ 0 & h^{\alpha\beta} \end{pmatrix}. \tag{2.11}$$

The potential (2.6) reads

$$V = -w\sum_{s\in S}\Lambda_s \mathrm{e}^{2U^{s}(x)} \tag{2.12}$$

where $U^{s}(x) = U^{s}_{A}x^{A}$ are defined as follows

$$U^{s} = U^{s}(\phi, \varphi) = \lambda_{s\alpha}\varphi^{\alpha} + \sum_{i=0}^{n} d_i\phi^{i}\,, \tag{2.13}$$

or in components

$$(U^{s}_{A}) = (d_i, \lambda_{s\alpha})\,. \tag{2.14}$$

The integrability of the Lagrange system (2.5) depends upon the scalar products of co-vectors U^s corresponding to $\bar{G}$:

$$(U, U') = \bar{G}^{AB} U_A U'_B \,. \tag{2.15}$$

These products have the following form

$$(U^s, U^{s'}) = -b + \lambda_s \cdot \lambda_{s'} \,, \tag{2.16}$$

where $s, s' \in S$ and

$$\lambda_s \cdot \lambda_{s'} \equiv \lambda_{s\alpha} \lambda_{s'\beta} h^{\alpha\beta} \,, \tag{2.17}$$

$$b = \frac{D-1}{D-2} \,. \tag{2.18}$$

We put the orthogonality restriction on the vectors U^s

$$(U^s, U^{s'}) = -b + \lambda_s \cdot \lambda_{s'} = 0 \,, \tag{2.19}$$

$s \neq s'$, and also consider non-degenerate case

$$(U^s, U^s) = -b + \lambda_s \cdot \lambda_s \neq 0 \tag{2.20}$$

(see (1.3) and (1.4)).

In what follows we denote

$$h_s = (U^s, U^s)^{-1} \equiv \frac{1}{\lambda_s^2 - b} \,. \tag{2.21}$$

The further consideration is based upon the orthogonality conditions assumed.

3. Classical solutions

Here we will integrate the Lagrange equations corresponding to the action (2.5) in the harmonic time gauge $\gamma = \gamma_0$. We get a lagrangian

$$L = \frac{1}{2} \bar{G}_{AB} \dot{x}^A \dot{x}^B - V \,, \tag{3.1}$$

where $(\bar{G}_{AB})$ and V are defined in (2.11) and (2.12), respectively. The zero-energy constraint reads:

$$E = \frac{1}{2} \bar{G}_{AB} \dot{x}^A \dot{x}^B + V = 0 \,. \tag{3.2}$$

The solutions to Lagrange equations corresponding to (3.1) in the orthogonal and non-degenerate case (2.19), (2.20) read (see [34, 19]):

$$x^A = -\sum_{s \in S} \frac{U^{sA}}{(U^s, U^s)} \ln |f_s| + c^A u + \bar{c}^A \,. \tag{3.3}$$

Here

$$f_s = R_s \ \mathrm{sh}\left(\sqrt{\mathrm{C_s}}(\mathrm{u}-\mathrm{u_s})\right), \quad \mathrm{C_s} > 0\,, \quad \epsilon_s < 0\,; \tag{3.4}$$

$$R_s \sin\left(\sqrt{|C_s|}(u-u_s)\right), \quad C_s < 0\,, \quad \epsilon_s < 0\,; \tag{3.5}$$

$$R_s \ \mathrm{ch}\left(\sqrt{\mathrm{C_s}}(\mathrm{u}-\mathrm{u_s})\right), \quad \mathrm{C_s} > 0\,, \quad \epsilon_s > 0\,; \tag{3.6}$$

$$\left|\frac{2\Lambda_s}{h_s}\right|^{1/2} (u-u_s)\,, \quad C_s = 0\,, \quad \epsilon_s < 0\,, \tag{3.7}$$

where u_s and C_s are constants and

$$R_s = |2\Lambda_s/(h_s C_s)|^{1/2}\,, \tag{3.8}$$

$$\epsilon_s = -\mathrm{sign}(w\Lambda_s h_s)\,. \tag{3.9}$$

Vectors $c = (c^A)$ and $\bar{c} = (\bar{c}^A)$ satisfy the linear constraint relations

$$U^s(c) = U^s_A c^A = \sum_{i=0}^{n} d_i c^i + \lambda_{s\alpha} c^\alpha = 0\,, \tag{3.10}$$

$$U^s(\bar{c}) = U^s_A \bar{c}^A = \sum_{i=0}^{n} d_i \bar{c}^i + \lambda_{s\alpha} \bar{c}^\alpha = 0\,. \tag{3.11}$$

The zero-energy constraint reads

$$E = \sum_{s\in S} E_s + \frac{1}{2}\bar{G}_{AB} c^A c^B = 0\,, \tag{3.12}$$

where $C_s = 2E_s(U^s, U^s)$, or, equivalently,

$$\sum_{s\in S} C_s h_s + h_{\alpha\beta} c^\alpha c^\beta + \sum_{i=0}^{n} d_i (c^i)^2 - \left(\sum_{i=0}^{n} d_i c^i\right)^2 = 0\,. \tag{3.13}$$

The solutions. Using (3.3) and relations for contravariant components $U^{sA} = \bar{G}^{AB} U^s_B$

$$U^{si} = G^{ij} U^s_j = -\frac{1}{D-2}\,, \quad U^{s\alpha} = \lambda^\alpha_s\,, \tag{3.14}$$

we are led to relations for the metric

$$g = \left(\prod_{s\in S} f_s^{2b\ h_s}\right) \left\{ \mathrm{e}^{2cu+2\bar{c}} w du \otimes du + \left(\prod_{s\in S} f_s^{-2h_s}\right) \sum_{i=0} \mathrm{e}^{2c^i u + 2\bar{c}^i} g^i \right\} \tag{3.15}$$

and scalar fields

$$\varphi^\alpha = -\sum_{s\in S} h_s \lambda^\alpha_s \ln|f_s| + c^\alpha u + \bar{c}^\alpha\,, \tag{3.16}$$

$\alpha = 1, \dots, l$.

Here

$$c = \sum_{i=0}^{n} d_i c^i\,, \qquad \bar{c} = \sum_{i=0}^{n} d_i \bar{c}^i\,. \tag{3.17}$$

The functions f_s are defined in (3.4)–(3.7) and the relations on the parameters of solutions c^A, $\bar{c}^A$ ($A = i, \alpha$) and C_s imposed in (3.10)–(3.11) and (3.13), respectively. (h_s are defined in (2.21).)

4. Special solutions

Now we consider a special case of classical solutions from the previous section when $C_s = u_s = c^i = c^\alpha = 0$ and

$$w\Lambda_s(\lambda_s^2 - b) > 0\,, \tag{4.1}$$

$s \in S$.

We get two families of solutions written in synchronous-type variable with:

A) power-law dependence of scale factors for $B \neq -1$,

B) exponential dependence of scale factors for $B = -1$,

where

$$B = B(\lambda) = b \sum_{s \in S} h_s\,. \tag{4.2}$$

Note that $h_s = h_s(\lambda_s) = (\lambda_s^2 - b)^{-1}$.

4.1 Power-law solutions

Let us consider the solution corresponding to the case $B \neq -1$. The solution reads (see appendix B)

$$g = w d\tau \otimes d\tau + \tau^{2\nu} \sum_{i=0}^{n} A_i g^i\,, \tag{4.3}$$

$$\varphi^\alpha = -\frac{1}{B+1} \sum_{s \in S} h_s \lambda_s^\alpha \ln \tau + \varphi_0^\alpha\,, \tag{4.4}$$

where $\tau > 0$,

$$\nu = \frac{B}{(B+1)(D-1)}\,, \tag{4.5}$$

and

$$|2\Lambda_s / h_s| \exp(2\lambda_{s\alpha}\varphi_0^\alpha) = (B+1)^{-2}\,, \tag{4.6}$$

$s \in S$; and $A_i > 0$ are arbitrary constants.

Solutions with "acceleration". Let us consider the cosmological case $w = -1$. We get an accelerated expansion of factor spaces if and only if $\nu > 1$ or, equivalently,

$$-b < B(\lambda) < -1\,. \tag{4.7}$$

Remark 1. *For $-1 < B = B(\lambda) < 0$ we get $\nu < 0$, that corresponds to a contraction of factor spaces. When $B < -b$ or $B > 0$ we are led to the expansion with deceleration, since $0 < \nu < 1$ in this case. The case $B = 0$ corresponds to static toy "universe". For $B = -b$ we obtain the expansion with zero acceleration ($\nu = 1$).*

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Let us consider the case when the matrix $(h_{\alpha\beta})$ is positive definite (e.g. $h_{\alpha\beta} = \delta_{\alpha\beta}$). It was shown in appendix D that the condition $B(\lambda) \neq -1$ in this case implies that vectors λ_s are linearly independent. It follows also from appendix D that the condition $B(\lambda) < -1$ implies that

$$\lambda_{s_0}^2 < b\,, \qquad \lambda_s^2 > b \text{ for all } s \neq s_0\,. \tag{4.8}$$

Here $s_0 \in S$. These relations combined with inequalities (4.1) lead to

$$\Lambda_{s_0} > 0\,, \qquad \Lambda_s < 0 \text{ for all } \ s \neq s_0\,, \tag{4.9}$$

i.e. one exponent term in potential should be positive and others should be negative.

4.2 Solutions with exponential scale factors

Here we consider the solution corresponding to the case $B = -1$. The solution reads (see appendix C)

$$g = w d\tau \otimes d\tau + \exp(2m\tau) \sum_{i=0}^{n} A_i g^i\,, \tag{4.10}$$

$$\varphi^\alpha = (D-1)m\tau \sum_{s\in S} h_s \lambda_s^\alpha + \varphi_0^\alpha\,, \tag{4.11}$$

where

$$|2\Lambda_s/h_s| \exp(2\lambda_{s\alpha}\varphi_0^\alpha) = m^2(D-1)^2\,, \tag{4.12}$$

$s \in S$; m is parameter and $A_i > 0$ are arbitrary constants.

In the cosmological case $w = -1$ we get an accelerated expansion of factor spaces if and only if $m > 0$.

Let us consider the case when $(h_{\alpha\beta})$ is positive definite. As it was shown in appendix D the condition $B(\lambda) = -1$ in this case implies that vectors λ_s are linearly dependent, the linear term in (4.11) vanishes and hence

$$\varphi^\alpha = \varphi_0^\alpha\,, \tag{4.13}$$

i.e. for $(h_{\alpha\beta})$ of euclidean signature all scalar fields are constants when exponential expansion is under consideration. We note that any subset of vectors $\lambda_s, s \in S \setminus \{s_0\}$, is linearly independent one in this case (see appendix D).

Remark 2. *It may be shown that for the solution (4.10)–(4.12) the kinetic term for scalar fields vanishes, i.e.* $h_{\alpha\beta} g^{MN} \partial_M \varphi^\alpha \partial_N \varphi^\beta = 0$ *and the scalar potential is constant:* $V_\varphi(\varphi) = V_\varphi(\varphi_0) \equiv \Lambda$ *and* $(-w)2\Lambda = m^2(D-1)(D-2)$ *(see (4.1)). For* $w = -1$ *and* $\Lambda > 0$ *the metric (4.10) is coinciding with that for multidimensional model from [31] with pure cosmological term* Λ*. Thus, the exponential expansion of factor spaces is driven by effective cosmological term* Λ *and is not sensitive to time dependence of scalar fields, that takes place only for non-euclidean signatures of* $(h_{\alpha\beta})$*.*

4.3 One-exponent example

The simplest case with one term in the potential (1.2) is interesting due to the absence of the restriction (2.19). It follows from eq. (4.1) that the potential is positive:

$$V_{\varphi} = \Lambda e^{2\lambda_{\alpha}\varphi^{\alpha}} , \quad \Lambda > 0 \tag{4.14}$$

(we discard the indices in this example).

The accelerated power law expansion takes place, if

$$0 < \lambda^2 < \frac{1}{D-2} , \tag{4.15}$$

i.e. λ^2 should be small enough but not equal to zero. The inequality is coinciding (up to notations) with the eq. (1.9) from [22]. In this case (4.5) reads as $\nu = (\lambda^2(D-2))^{-1}$.

For the exponential expansion eqs. (4.1) and $B(\lambda) = -1$ imply

$$\lambda^2 = 0 , \tag{4.16}$$

i.e. λ should be zero vector for positive definite matrix $(h_{\alpha\beta})$ or light-like vector for $(h_{\alpha\beta})$ of pseudo-euclidean signature.

4.4 Generalized isotropization

In the case of one exponent the special solutions considered above are attractors in the limit $\tau \to +\infty$ (τ is "synchronous" variable) for general solutions with sinh-dependent function f_s (see (3.4)), when $\epsilon_s < 0$, or, equivalently, (4.1) is satisfied and $B \leq -1$. This fact may be readily verified using the relation $f_s \sim (u - u_s)$ for $u \to u_s$.

Here we consider a more general case of this attractor behavior for multi-exponent case. Let S_- be a subset of all S satisfying $\epsilon_s < 0$, or, equivalently, relations (4.1). Let us study a family of solutions with positive C_s and hence with sinh-dependence of f_s for $s \in S_-$ and cosh-dependence of f_s for $s \in S \setminus S_-$. Consider the solutions on the interval $(u_*, +\infty)$, where

$$u_* = \max(u_s, s \in S_-) . \tag{4.17}$$

We denote by S_* a subset of all $s \in S_-$ obeying $u_s = u_*$. All functions $f_s(u - u_s)$ are smooth on this interval and near u_* they behave as $f_s \sim (u - u_*)$ for $s \in S_*$, or as (non-zero) constants for $s \in S \setminus S_*$.

Introducing the synchronous time variable by relation

$$\tau = \int du \exp(cu + \bar{c}) \prod_s (f_s(u - u_s))^{bh_s} + \mathrm{const} \tag{4.18}$$

we get $\tau \to +\infty$ as $u \to u_*$ in two cases:

A) $B_* < -1$ and

$$\tau \sim (u - u_*)^{B_* + 1} ; \tag{4.19}$$

B) $B_* = -1$ and

$$\tau \sim -\ln|u - u_*| . \tag{4.20}$$

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Here we denote

$$B_* = B(\lambda, S_*) = \sum_{s \in S_*} h_s b \,. \tag{4.21}$$

Case A. Using arguments analogous to those presented in appendix B we get the following asymptotical relations in the limit $\tau \to +\infty$ for $B_* < -1$

$$g_{as} = w d\tau \otimes d\tau + \sum_{i=0}^{n} A_i \tau^{2\nu_*} g^i \,, \tag{4.22}$$

$$\varphi^\alpha_{as} = -\frac{1}{B_* + 1} \sum_{s \in S_*} h_s \lambda^\alpha_s \ln \tau + \varphi^\alpha_0 \,, \tag{4.23}$$

where

$$\nu_* = \frac{B_*}{(B_* + 1)(D - 1)} \,, \tag{4.24}$$

and $A_i > 0$ are constants.

Thus, we get an isotropization behaviour of scale factors for $\tau \to +\infty$, that is supported by exponential terms in the potential labelled by $s \in S_*$. Other exponential terms do not contribute to the power-law index ν_*.

For

$$-b < B_* < -1 \tag{4.25}$$

we obtain the accelerated expansion for large enough values of τ.

Case B. An analogous consideration in the case $B_* = -1$ leads to the following asymptotical relations in the limit $\tau \to +\infty$ (see also appendix C)

$$g_{as} = w d\tau \otimes d\tau + \sum_{i=0}^{n} A_i \exp(2m\tau) g^i \,, \tag{4.26}$$

$$\varphi^\alpha_{as} = (D - 1) m\tau \sum_{s \in S_*} h_s \lambda^\alpha_s + \varphi^\alpha_0 \,, \tag{4.27}$$

where $m > 0$ and

$$|2\Lambda_s / h_s| \exp(2\lambda_{s\alpha} \varphi^\alpha_0) = m^2 (D - 1)^2 \,, \tag{4.28}$$

$s \in S_*$. Thus, we obtain the accelerated expansion for large enough values of τ.

This is another regime of isotropization behaviour (for one-exponent case with $\lambda_s = 0$ see also [31, 32]).

Remark 3. *It may be shown that for the asymptotical solution (4.26)–(4.28) the kinetic term for scalar fields asymptotically vanishes, i.e.* $h_{\alpha\beta} g^{MN} \partial_M \varphi^\alpha \partial_N \varphi^\beta \to 0$ *as* $\tau \to +\infty$ *and the scalar potential is asymptotically constant:*

$$V_\varphi(\varphi) \to \sum_{s \in S_*} \Lambda_s \exp[2\lambda_s(\varphi_0)] \equiv \Lambda_* \tag{4.29}$$

as $\tau \to +\infty$*. Here* $(-w) 2\Lambda_* = m^2 (D - 1)(D - 2)$*. For* $w = -1$ *and* $\Lambda_* > 0$ *the "asymptotical" metric (4.10) is coinciding with that for multidimensional model from [31] with pure cosmological term* Λ_**. When* $(h_{\alpha\beta})$ *has euclidean signature the exponential isotropization behaviour takes place only for* $S = S_*$ *(it follows from appendix D). In this case* $\varphi_{as} = \varphi_0$*.*

4.5 Kasner-type behaviour

Let us study the solution with sinh-dependence in the limit $u \to \infty$. We restrict ourselves by Kasner-like behaviour of solutions for small τ. The functions (3.4) and (4.18) behave as

$$f_s \sim e^{\sqrt{C_s}u}, \qquad \tau \sim e^{(b\Sigma+c)u}, \qquad u \to +\infty, \tag{4.30}$$

where $\Sigma = \sum_{s\in S} h_s\sqrt{C_s}$. To get the limit $\tau \to +0$ we put

$$b\Sigma + c < 0. \tag{4.31}$$

The metric and scalar fields in the limit $\tau \to +0$ read as follows:

$$g_{as} = w d\tau \otimes d\tau + \sum_{i=0}^{n} \tilde{A}_i \tau^{2\alpha^i} g^i, \tag{4.32}$$

$$\varphi^\beta_{as} = \alpha^\beta \ln\tau + \tilde{\varphi}^\beta_0, \tag{4.33}$$

where the Kasner parameters are given by the formulas

$$\alpha^i = \frac{\Sigma\cdot(D-2)^{-1} + c^i}{b\Sigma + c}, \tag{4.34}$$

$$\alpha^\beta = \frac{-\sum_{s\in S} h_s\sqrt{C_s}\lambda^\beta_s + c^\beta}{b\Sigma + c}. \tag{4.35}$$

One can see that these parameters obey the generalized Kasner relations:

$$\sum_{i=0}^{n} d_i\alpha^i = 1, \qquad \sum_{i=0}^{n} d_i(\alpha^i)^2 + h_{\alpha\beta}\alpha^\alpha\alpha^\beta = 1. \tag{4.36}$$

We note that for $b\Sigma + c < 0$ we get a Kasner-like asymptotics in the limit $\tau \to -\infty$ (with τ replaced by $|\tau|$ in (4.32) and (4.33)).

4.5.1 Example: two spaces with one scalar field

As an example let us consider the model with two factor spaces, one scalar field and one term in potential. It is not difficult to verify, that the parameters

$$\alpha^0 = \frac{(d_0 - R)}{d_0(d_0+d_1)}, \qquad \alpha^1 = \frac{(d_1 + R)}{d_1(d_0+d_1)}, \qquad \alpha^\beta = 0, \tag{4.37}$$

where $R = \sqrt{d_0 d_1 (D-2)}$, satisfy the eqs. (4.36).

For $d_1 = 3$, $d_2 = 6$ the behaviour of scale factors are presented in figure 1, when: a) $\lambda^2 = 1/16$ (asymptotically power law expansion); b) $\lambda = 0$ (asymptotically exponential expansion).

Remark 4. *It will be shown in a separate publication that Kasner-like behaviour near the singularity (as $\tau \to +0$) is a generic one for $n > 0$ and a wide class of scalar potentials (1.2).*

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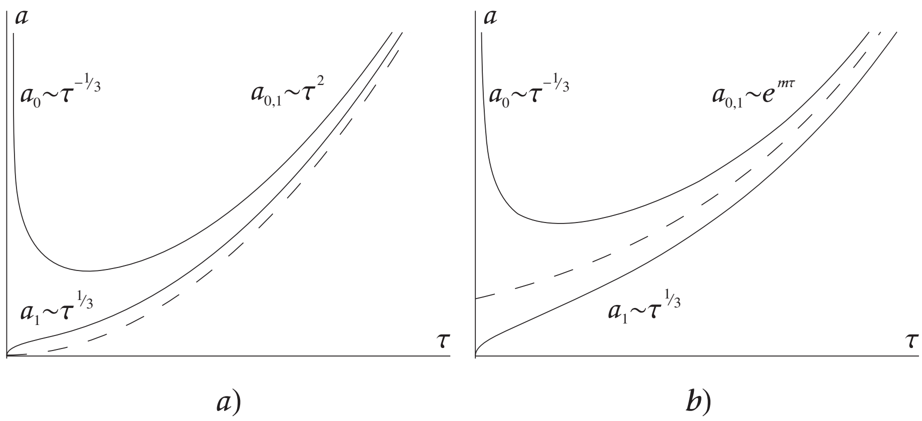

Figure 1: The behaviour of scale factors of two factor spaces of dimensions $d_0 = 3$ and $d_1 = 6$ in the presence of one scalar field and one term in potential for the cases: a) $\lambda^2 = 1/16$; b) $\lambda = 0$. In both cases the Kasner parameter for the scalar field is chosen to be zero.

5. Quantum solutions

Here we find the quantum analogues of the classical solutions presented above.

5.1 General solutions

The standard quantization of the energy constraint $\mu E = 0$, where $\mu \neq 0$ is a parameter, leads to the Wheeler-DeWitt (WDW) equation (see, for example, [33, 37, 38]) in the harmonic-time gauge $\gamma = \gamma_0(\phi)$

$$\hat{H}\Psi \equiv \left(-\frac{1}{2\mu}\Delta[\bar{G}] + \mu V\right)\Psi = 0\,. \tag{5.1}$$

Here V is potential defined in (2.12), $\Delta[\bar{G}]$ is the Laplace operator, corresponding to $\bar{G}$, and Ψ is wave function.

The minisuperspace metric may be diagonalized by the linear transformation

$$z^{\bar{A}} = S^{\bar{A}}{}_{B} x^{B}\,, \qquad (z^{\bar{A}}) = (z^0, z^a, z^s) \tag{5.2}$$

as follows

$$\bar{G}_{AB} dx^A \otimes dx^B = -dz^0 \otimes dz^0 + \sum_{s \in S} \eta_s dz^s \otimes dz^s + dz^a \otimes dz^b \eta_{ab}\,, \tag{5.3}$$

where $a, b = 1, \ldots, n + l - |S|$; $\eta_{ab} = \eta_{aa}\delta_{ab}$; $\eta_{aa} = \pm 1$, $\eta_s = \mathrm{sign}(h_s)$, and

$$q_s z^s = U^s(x)\,, \tag{5.4}$$

with

$$q_s = |h_s|^{-1/2} = \sqrt{|\lambda_s^2 - b|} > 0\,, \tag{5.5}$$

$s \in S$.

We are seeking the solution to WDW equation (5.1) by the method of the separation of variables, i.e. we put

$$\Psi = \left(\prod_{s \in S} \Psi_s(z^s) \right) \mathrm{e}^{\mathrm{i} p_a z^a} . \tag{5.6}$$

The wave function (5.6) satisfies to WDW equation (5.1) if

$$2\hat{H}_s \Psi_s \equiv \left\{ -\eta_s \frac{\partial}{\partial z^s} \left(\frac{\partial}{\partial z^s} \right) - 2w \Lambda_s \mathrm{e}^{2 q_s z^s} \right\} \Psi_s = 2 \mathcal{E}_s \Psi_s \,, \tag{5.7}$$

where

$$\eta^{ab} p_a p_b + 2 \sum_{s \in S} \mathcal{E}_s = 0 \,. \tag{5.8}$$

(compare with classical relation (3.12)).

The linearly independent solutions to eq. (5.7) reads

$$\Psi_s(z^s) = B^s_{\omega_s} \left(\sqrt{-2w \eta_s \Lambda_s} \, \frac{\mathrm{e}^{q_s z^s}}{q_s} \right) , \tag{5.9}$$

where

$$\omega_s = \sqrt{-2 \mathcal{E}_s h_s} \,, \tag{5.10}$$

$s \in S$ and $B^s_\omega = a_s I_\omega + b_s K_\omega$ are superpositions of modified Bessel function.

The general solution to the WDW equation (5.1) is a superposition of the "separated" solutions (5.6).

5.2 Special solutions

Let us consider the special "ground state" solutions with $p_a = \mathcal{E}_s = 0$ in (5.9), when relations (4.1) are imposed. These solutions read

$$\Psi = \prod_{s \in S} \left(a_s J_0 (\sqrt{2|h_s \Lambda_s|} \; v \mathrm{e}^{\lambda_s(\varphi)}) + b_s H^{(1)}_0 (\sqrt{2|h_s \Lambda_s|} \; v \mathrm{e}^{\lambda_s(\varphi)}) \right) , \tag{5.11}$$

where J_0 and $H^{(1)}_0$ are the Bessel and Hankel functions, respectively, and $v = \exp(\sum\limits_{i=0}^{n} d_i \phi^i)$ is a volume scale factor. These quantum solutions correspond to special classical solutions considered in the previous section.

We note that for large values of

$$v_s = \sqrt{2|h_s \Lambda_s|} \; v \exp(\lambda_s(\varphi)) \tag{5.12}$$

we get a quasi-classical regime, that may be obtained (along a line as it was done in [39]) using asymptotical relations for Bessel functions. For small values of "quasivolumes" v_s we are led to a quantum regime. The crucial point here is that the quantum domain is not defined only by volume scale factor v, but by "quasivolumes" v_s, depending also upon scalar fields. In strong enough scalar fields with certain signs (and the direction of vector φ) and/or small $|h_s \Lambda_s|$ one can obtain a quantum behaviour of a wave function for big enough values of volume scale factor v. Analogous "effect" takes place also for quantum solutions with "perfect fluid" [16] when certain equations of state are adopted.

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6. Conclusions

Here we obtained a family of multidimensional cosmological solutions with $(n+1)$ Ricci-flat spaces in the theory with several scalar fields and multiple exponential potential.

The classical and quantum solutions are obtained if the (orthogonality and non-degeneracy) relations (2.19) and (2.20) on U-vectors are imposed, or, equivalently, when relations (1.3) and (1.4) on coupling vectors are satisfied. These solutions in fact correspond to $A_1 + \cdots + A_1$ Toda-like solutions (for perfect fluid case see [34]).

Here we singled out the solutions with power-law and exponential behaviours of scale factors. We proved that power-law dependence may take place only when coupling vectors λ_s are linearly independent; exponential dependence may occur only for a linearly dependent set of λ_s obeying the condition $B(\lambda) = -1$. We obtained the restriction on coupling vectors (4.7) that cuts power-law solutions with acceleration. In subsection 4.3 the generalized isotropization behavior of certain class of general solutions was found. Any asymptotics with power-law or exponential behavior corresponds to a certain subset of exponential terms in the potential (all other exponential terms do not contribute to this asymptotical behaviour).

In the quantum case we solved the Wheeler-DeWitt equation and singled out special "ground state" solutions that are quantum analogous of special classical solutions from section 4. These solutions are defined as products of Bessel functions and depend upon a set of quasivolumes that may be small enough (i.e. belonging to "quantum domain") even when the volume scale factor of the toy "universe" is in the classical region (in the Planck scale).

Acknowledgments

This work was supported in part by the Russian Ministry of Science and Technology, Russian Foundation for Basic Research (RFFI-01-02-17312-a) and Project DFG (436 RUS 113/678/0-1(R)).

A. Equations of motion

Here we outline for the sake of completeness the equations of motions corresponding to the action (1.1)

$$\mathcal{R}_{MN} - \frac{1}{2} g_{MN} \mathcal{R} = T_{MN} \,, \tag{A.1}$$

$$\triangle[g]\varphi^\alpha - \sum_{s \in S} 2\lambda_s^\alpha e^{2\lambda_s(\varphi)} \Lambda_s = 0 \,. \tag{A.2}$$

In (A.1)

$$T_{MN} = h_{\alpha\beta} \left(\partial_M \varphi^\alpha \partial_N \varphi^\beta - \frac{1}{2} g_{MN} \partial_P \varphi^\alpha \partial^P \varphi^\beta \right) - V_\varphi g_{MN} \,. \tag{A.3}$$

B. Power-law expansion

Here we derive the solution (4.3)–(4.6) from the general one. We start from the relation (3.3) that can be written in our case as follows

$$x^A(u) = -\sum_{s\in S} h_s U^{sA} \ln(\sqrt{|2\Lambda_s/h_s|}u) + \bar{c}^A \,, \tag{B.1}$$

where $u > 0$.

Introducing a new variable $\tau > 0$ by formula

$$u = C\tau^{1/(B+1)} \,, \qquad C > 0 \,, \tag{B.2}$$

we rewrite (B.1) in the following manner

$$x^A = -\sum_{s\in S} h_s U^{sA} \ln\tau + x_0^A \tag{B.3}$$

with constants

$$x_0^A = \bar{c}^A - \sum_{s\in S} h_s U^{sA} \ln\left(\sqrt{|2\Lambda_s/h_s|}C\right) . \tag{B.4}$$

Due to orthogonality of U^s-vectors the constraints on integration constant $U^s_A \bar{c}^A = 0$ may be written in the equivalent form

$$U^s_A x_0^A = -\ln(\sqrt{|2\Lambda_s/h_s|}C) \,. \tag{B.5}$$

In components the solution (B.3) for $(x^A) = (\phi^i, \varphi^\alpha)$ reads

$$\phi^i = \frac{B}{(B+1)(D-1)} \ln\tau + \phi_0^i \,, \tag{B.6}$$

$$\varphi^\alpha = -\frac{1}{B+1} \sum_{s\in S} h_s \lambda_s^\alpha \ln\tau + \varphi_0^\alpha \,. \tag{B.7}$$

Here $(x_0^A) = (\phi_0^i, \varphi_0^\alpha)$.

For $\gamma_0(\phi) = \sum\limits_{i=0}^{n} d_i \phi^i$ we get

$$\gamma_0(\phi) = \frac{B}{(B+1)} \ln\tau + \phi_0 \,, \tag{B.8}$$

where $\phi_0 = \sum\limits_{i=0}^{n} d_i \phi_0^i$.

From the definition of "synchronous" variable

$$\exp(2\gamma_0(\phi)) du^2 = d\tau^2 \tag{B.9}$$

we obtain

$$C = |B+1| \exp(-\phi_0) \tag{B.10}$$

and hence (B.5) reads

$$\lambda_{s\alpha} \varphi_0^\alpha = -\ln\left(\sqrt{\left|\frac{2\Lambda_s}{h_s}\right|}C\right) . \tag{B.11}$$

The solution (4.3)–(4.6) follows from the formulas (B.6), (B.7), (B.9), (B.11) and $A_i = \exp(2\phi_0^i)$.

C. Exponential expansion

The solution (4.10)–(4.12) may be obtained just along a line as it was done for the power-law case. The only difference here is the relation

$$u = C\exp(M\tau)\,, \qquad C > 0\,, \tag{C.1}$$

instead of (B.2). Using a procedure analogous to considered hereabove we get

$$\phi^i = -\frac{1}{D-1}M\tau + \phi_0^i\,, \tag{C.2}$$

$$\varphi^\alpha = -\sum_{s\in S} h_s\lambda_s^\alpha M\tau + \varphi_0^\alpha \tag{C.3}$$

and

$$\gamma_0(\phi) = -M\tau + \phi_0\,. \tag{C.4}$$

From (B.9) we get

$$C = |M|^{-1}\exp(-\phi_0) \tag{C.5}$$

and introducing new parameter $m = -M/(D-1)$ we obtain the solution (4.10)–(4.12).

D. b-proper set of vectors

The statements of this subsection are based on relations (1.3) and (1.4)), that are used in Definition of b-proper set (see below). The conditions for exponential expansion follow from Theorem. Lemma 2 is useful for analysis of "power-law" expansion.

In what follows $\{\lambda_i\}$ means a set of vectors $\lambda_1,\ldots,\lambda_m \in \mathbb{R}^l, l\in\mathbb{N}$. The vector space $\mathbb{R}^l$ is equipped with a scalar product (it corresponds to the positive definite matrix $(h_{\alpha\beta})$ in the bulk of the article).

Definition 1. *Let $b > 0$. The set of vectors $\{\lambda_i\}$ is called a b-proper one if*

$$\lambda_i\cdot\lambda_j = b\,, \tag{D.1}$$

$$\lambda_i^2 \neq b\,, \tag{D.2}$$

$i \neq j,\ i,j = 1,\ldots m.$

Lemma 1. *Let $K = (\lambda_i\cdot\lambda_j)$ be a matrix of scalar products for a b-proper set of vectors $\{\lambda_i\}$. Then*

$$\det K = (B+1)\prod_{i=1}^{m}(\lambda_i^2 - b)\,, \tag{D.3}$$

where

$$B = B(\lambda_1,\ldots,\lambda_m) \equiv \sum_{i=1}^{m}\frac{b}{\lambda_i^2 - b}\,. \tag{D.4}$$

Proof. Using relations

$$K_{ij} = \lambda_i \cdot \lambda_j = (\lambda_i^2 - b)\delta_{ij} + b = (\lambda_i^2 - b)\left(\delta_{ij} + \frac{b}{\lambda_i^2 - b}\right) \tag{D.5}$$

we represent the matrix K as a product of a matrix $\bar{K}$ and a diagonal matrix D:

$$\bar{K}_{ij} = \delta_{ij} + \frac{b}{\lambda_i^2 - b}\,, \qquad D_{ij} = \delta_{ij}(\lambda_i^2 - b)\,. \tag{D.6}$$

A product of determinants $\det D$ and $\det \bar{K}$ yields (D.3). Here we used the relation $\det \bar{K} = B + 1$, that can be readily proved.

Theorem 1. *Let $\{\lambda_i\}$ be a b-proper set. The set $\{\lambda_i\}$ is linearly dependent if, and only if*

$$B = B(\lambda_1, \dots, \lambda_m) = -1\,. \tag{D.7}$$

In this case

$$\sum_{i=1}^{m} \frac{\lambda_i}{\lambda_i^2 - b} = 0 \tag{D.8}$$

and any subset of $m-1$ vectors from $\{\lambda_i\}$ is linearly independent one.

Proof. The linear dependence of vectors $\{\lambda_i\}$ is equivalent to $\det K = 0$ that is equivalent (due to Lemma 1) to relation (D.7).

From the relation

$$B(\lambda_1, \dots, \lambda_m) = B(\lambda_1, \dots, \lambda_{m-1}) + \frac{b}{\lambda_m^2 - b} = -1 \tag{D.9}$$

we get $B(\lambda_1, \dots, \lambda_{m-1}) \neq -1$ and hence the vectors $\lambda_1, \dots, \lambda_{m-1}$ are linearly independent. It is obvious that any $m-1$ λ-vectors are also linearly independent.

To prove (D.8) let us denote

$$\lambda = \sum_{i=1}^{m} \frac{\lambda_i}{\lambda_i^2 - b}\,. \tag{D.10}$$

Using the scalar products (D.5) we obtain

$$\lambda^2 = \frac{B(B+1)}{b} = 0 \tag{D.11}$$

and, hence, $\lambda = 0$. The Theorem is proved.

As consequence we obtain the inequality on the number of b-proper vectors: $m \le l+1$. (The equality $m = l+1$ takes place when the vectors are linearly dependent.)

Lemma 2. *In the b-proper set $\{\lambda_i\}$:*

a there are no two or more vectors λ_i obeying $\lambda_i^2 < b$;

b $B > -1$ $\Leftrightarrow$ vectors λ_i are linearly independent and all $\lambda_i^2 > b$.

Proof.

a Let us suppose that there are two vectors $\lambda_i,\ \lambda_j$, such that $\lambda_i^2 < b$ and $\lambda_j^2 < b$. Then $b^2 = (\lambda_i \cdot \lambda_j)^2 \leq \lambda_i^2 \lambda_j^2 < b^2$, i.e. we come to a contradiction.

b If $B > -1$ then by the Theorem the vectors are linearly independent and hence the matrix $(\lambda_i \cdot \lambda_j) = (K_{ij})$ is positive definite. This implies $\det K > 0$. Due to relation (D.3) and part a. of this lemma we get: $\lambda_i^2 > b$ for all vectors.

Now, let the vectors $\{\lambda_i\}$ be linearly independent and all $\lambda_i^2 > b$. Then $\det K > 0$, and due to (D.3), $B + 1 > 0$. The lemma is proved.

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